Appendix A
Notation

Scalars, Vectors, and Matrices

Scalars Scalars are denoted by plain (not boldface) characters, such as x, a, i, μ.

Vectors Vectors are denoted by boldface characters, so, for example, $\mathbf{x} = (x_1, x_2, \dots, x_n)$.

Matrices Matrices are denoted by boldface uppercase characters, such as $\mathbf{A} = [a_{ij}]$ where a_{ij} denotes the element in the i^{th} row and j^{th} column of **A**. The i^{th} row of a matrix **A** is denoted $\mathbf{A}^i$, and the j^{th} column is denoted $\mathbf{A}_j$.

Inner products The inner product of two vectors **x** and **y** is denoted $\mathbf{x}^\top \mathbf{y}$ and is defined by

$$\mathbf{x}^\top \mathbf{y} = \sum_{i=1}^{n} x_i y_i.$$

The inner product of a matrix **A** and a vector **x** is denoted $\mathbf{A}^\top \mathbf{x}$ and is defined as the vector

$$\mathbf{A}^\top \mathbf{x} = (\mathbf{A}^{1\top} \mathbf{x}, \mathbf{A}^{2\top} \mathbf{x}, \dots, \mathbf{A}^{n\top} \mathbf{x}).$$

The following is a list of variables along with a description of their typical meanings throughout the text.

Roman Variables

A Incidence matrix for a network model $\mathbf{A} = [a_{ij}]$, where $a_{ij} = 1$ if resource i is used by product j and $a_{ij} = 0$ otherwise; m rows, n columns.

$\mathbf{A}^i$ The i^{th} row of the incidence matrix **A**.

A^i The set of products that use resource i.

$\mathbf{A}_j$ The j^{th} column of the incidence matrix **A**. Also used to denote the set of resources used by product j.

A_j The set of resources used by product j.

$B_j(y, D)$ The j^{th} "fill event."

b_j Booking limit or nested booking limit.

$c, c(x)$ Variable cost of production; cost function. Used in economics and overbooking models

C_i, **C** Initial capacity of resource i; vector of initial capacities. Also used to denote the j^{th} *complete set,* $C_j = \{1, \ldots, j\}$.

d_j, **d**, $d(p)$, **d**(**p**) Demand (deterministic or mean) for product j; vector of demands. A demand function depending on price p; vector demand function.

D_j, **D** Demand (random variable) for product j; vector of demand random variables.

h_{ij}, **h** Cost parameters or vector of cost parameters in an overbooking models.

i Generally indexes resources but also used as a generic index.

j Generally indexes products but also used as a generic index.

$J(p), J(v)$ The marginal revenue as a function of price; the virtual value of a buyer with value v.

k Capacity cost in economics models; generic integer variable.

m The number of resources; generic integer variable.

n The number of products; generic integer variable.

N Population size or market potential in a pricing or an auction model.

$\mathcal{N}$ Denotes the set $\{1, 2, \ldots, n\}$ (e.g., set of n choice alternatives).

$p_j(t)$, **p**(t), p_j, **p** Price of product j at time t or vector of prices at time t; static price of product j; vectors of static prices.

q_j, q_t, **q** The probability that a customer shows up (e.g., the probability that class j does not cancel); vectors of probabilities.

$R(v)$ Expected revenue in an auction for buyer with value v.

S, S_k A subset of product classes or alternatives in a choice model; also used to represent a sum of random variables.

t Used to index time, either in discrete or continuous time.

T The number of periods in a discrete-time problem or the length of the horizon in a continuous-time problem. Also used to denote a generic set.

u_j, **u**, **u**(t), $u(x)$ Control variables in a dynamic program or other optimization problem, most often an accept or deny decision or a quantity decision. Also, u_j is used to denote the mean of a random-utility U_j in a random-utility model or to denote a utility function in economics models as in $u(x)$ is the utility of x.

U_j, **U** Random utility (random variable); vector of random utilities.

v_j, **v** Reservation price (private value) of customer j; vector of reservation price (private values).

$V_j(x), V_t(x)$ Optimal value function.

$V_t^M(\mathbf{x})$ A given approximation M to the optimal value function (e.g., $V_t^{DLP}(x)$ is the approximation of the value function produced by the deterministic linear program (DLP) model).

x_i, **x** Capacity variable; vector of capacities. For example, the remaining capacity of resource i in a dynamic program or the quantity of capacity chosen by firm i. Also used as the decision variable in overbooking models, where it represents the overbooking limit (virtual capacity). Vector of such state variables or capacities. Finally, used as capacity- or quantity-choice variable in economic models.

$y, y_j, \mathbf{y}$ Allocation variable or protection level for product j; vector of allocations or protection levels. Used in models for finding partitioned or nested allocations. Also the state variable (number of reservations on hand) in overbooking models.

z_t Notation used in forecasting. Data value of a forecast observed at time t (realization of random variable Z_t).

$\hat{z}_t$ Notation used in forecasting. Forecast (point estimate) of time-series value at time t (estimate of unrealized value Z_t).

Z_t Notation used in forecasting. The t^{th} random variable in a time series $Z_1, Z_2, \ldots$.

$Z(x), Z(y)$ Number of customers who show up (number of survivals) from a given number x, y of reservations on hand. Used in overbooking models.

$\bar{Z}(x)$ Number of customers who cancel from a given number x of reservations on hand; $\bar{Z}(x) = x - Z(x)$.

Greek Variables

λ, λ_j An arrival rate in a deterministic demand model and arrival intensity or arrival probability in a probabilistic-demand model.

Δ The first-difference operator; if $g(x)$ is a function, then $\Delta g(x) = g(x) - g(x-1)$.

$\epsilon(p), \epsilon_{ij}(\mathbf{p})$ The elasticity of demand; the cross-price elasticity of demand for product i with respect to the price of product j.

μ The mean of a random variable.

$\Omega, \Omega_p, \Omega_d$ A constraint set; the contraint set of prices p and demand rates d.

$\pi_i, \pi_i(x), \pi$ A bid price value or function—or a dual price from a math program.

σ The variance of a random variable.

θ A generic parameter of a distribution or a scaling parameter.

$\Phi(z)$ The standard normal distribution (i.e., $\Phi(z) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-z/2} dz$).

$\phi(z)$ The standard normal density (i.e., $\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z}{2}}$).

$\psi_X(t)$ The moment-generating function of a random variable *X*.

ω An elementary outcome in a probability space (e.g., a random variable is $X(\omega)$).

Miscellaneous Symbols and Notation

$\Re, \Re_+, \Re^n, \Re^n_+$ The set of real numbers $(+\infty, +\infty)$; the set of nonnegative real numbers $[0, +\infty)$; the n-dimensional real plane and the n-dimensional positive orthant.

$\mathcal{Z}$ The set of integers, $\{\ldots,-2,-1,0,1,2,\ldots\}$.

$\mathbf{x}^\top, \mathbf{A}^\top$ The transpose of a vector x or a matrix **A**.

$x^+, (a-b)^+$ The positive part of x equal to $\max\{0, x\}$; the positive part of the quantity $(a-b)$.

$x^-, (a-b)^-$ The negative part of x equal to $\max\{0, -x\}$; the negative part of the quantity $(a-b)$.

e_j The j^{th} unit vector; a vector with one in the j^{th} component and zero in all other components.

$\mathbf{x}_{-j}$ The vector $\mathbf{x}$ without the j^{th} component; that is, the vector $\mathbf{x}_{-j} = (x_1, \ldots, x_{j-1}, x_{j+1}, \ldots, x_n)$.

C^1, C^2 The class of continuously differentiable functions on $\Re^n$; the class of all twice-continuously differentiable functions on $\Re^n$.

Abbreviations

a.s. Almost surely.

c.d.f. Cumulative distribution function.

i.i.d. Independent and identically distributed.

p.d.f. Probability-density function.

p.m.f. Probability mass function.